National 5 Christmas Problems

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Questions

1. Christmas Crackers (Quick Fire)

Solve the following problems:

(a) Factorise fully: $x^2 + 13x + 42$.

(b) Evaluate $\sqrt{28} + 4\sqrt{7}$, giving your answer as a surd in its simplest form.

(c) Calculate $3\frac{2}{7} - 2\frac{3}{7}$. Give your answer as a single fraction.

(d) Simplify the expression, leaving your answer in index form:

$$\frac{6^2 \times 6^0 \times (6^4)^3}{6^7}$$

(e) Determine the coordinates of the turning point of the parabola with equation $y = x^2 - 12x + 43$.

2. Present Delivery

Father Christmas delivered 3.2 billion presents in 2024. This represented a 3% increase on the number of presents delivered in 2023.
Calculate how many presents were delivered in 2023.
Round your answer to 3 significant figures.

3. Sleigh Queen

Mrs Claus takes the sleigh out for a test flight. The sleigh travels at an average speed of 3.2×10^3 mph.
She flies from the North Pole to Scotland, a distance of approximately 2,500 miles.
Calculate the time taken for this journey.
Give your answer in minutes, rounded to the nearest minute.

4. Naughty or Nice Statistics

Father Christmas gives a "Goodness Score" to each child, ranked from -10 (very naughty) to 10 (very nice). A sample of scores from 6 students at Newbattle High School is shown below:

$$5, \quad 8, \quad -2, \quad 6, \quad 9, \quad 4$$

(a) Calculate the mean and standard deviation of these scores.

(b) The population of all students in Scotland has a mean score of 3.2 and a standard deviation of 4.5.
Make two valid comparisons between the Newbattle sample and the Scottish population.

(c) It is discovered that the score of -2 actually belonged to a teacher, not a student! This score is removed from the sample.
State how removing this score will affect:

- The mean
- The standard deviation

Justify your answer (calculation not required).

5. The 12 Days of Christmas

On the 12th day of Christmas, my true love sent to me: *12 Drummers Drumming, 11 Pipers Piping, ..., and a Partridge in a Pear Tree.*
This means on Day 1, you get 1 gift. On Day 2, you get $1 + 2 = 3$ gifts. On Day 3, you get $1 + 2 + 3 = 6$ gifts.

(a) Complete the table below to show the pattern of gifts received.

Day (n)	**Gifts Received on this Day**	**Total Gifts Received So Far (T_n)**
1	1	1
2	$1 + 2 = 3$	$1 + 3 = 4$
3	$1 + 2 + 3 = 6$	$1 + 3 + 6 = 10$
4		
5		
...	...	...
12		

(b) Calculate the total number of gifts received over the full 12 days.

Solutions

1. Christmas Crackers

(a) **Factorise:** Find factors of 42 that add to 13: (6 and 7).

$$(x+6)(x+7)$$

(b) **Surds:** $\sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$.

$$2\sqrt{7} + 4\sqrt{7} = 6\sqrt{7}$$

(c) **Fractions:** Convert to improper fractions: $3\frac{2}{7} = \frac{23}{7}$, $2\frac{3}{7} = \frac{17}{7}$.

$$\frac{23}{7} - \frac{17}{7} = \frac{6}{7}$$

(Alternative method: $3 - 2 = 1$. $\frac{2}{7} - \frac{3}{7} = -\frac{1}{7}$. $1 - \frac{1}{7} = \frac{6}{7}$).

(d) **Indices:** Numerator: $6^2 \times 1 \times 6^{12} = 6^{14}$.

$$\frac{6^{14}}{6^7} = 6^{14-7} = 6^7$$

(e) **Turning Point (Completing the Square):** $y = (x-6)^2 - 36 + 43$ $y = (x-6)^2 + 7$
Turning point is at $(\mathbf{6}, \mathbf{7})$.

2. Present Delivery (Reverse Percentage)

Let x be the amount in 2023.

$$103\% : 3.2 \text{ billion}$$

$$1.03x = 3.2 \times 10^9$$

$$x = \frac{3.2 \times 10^9}{1.03} = 3,106,796,116...$$

Answer: $\mathbf{3.11 \times 10^9}$ or **3.11 billion** (3 s.f.).

3. Sleigh Queen

$$T = \frac{D}{S} = \frac{2500}{3.2 \times 10^3} = \frac{2500}{3200}$$

$$T = 0.78125 \text{ hours}$$

Convert to minutes: $0.78125 \times 60 = 46.875$ minutes. Answer: **47 minutes** (to nearest minute).

4. Naughty or Nice Statistics

(a) **Mean:** $\bar{x} = \frac{5+8+(-2)+6+9+4}{6} = \frac{30}{6} = \mathbf{5}$. **Standard Deviation:**

x	$(x-\bar{x})^2$
5	0
8	9
-2	49
6	1
9	16
4	1
$\sum$	76

$$s = \sqrt{\frac{76}{6-1}} = \sqrt{\frac{76}{5}} = \sqrt{15.2} \approx \mathbf{3.90}$$

(b) **Comparisons:** 1. On average, the Newbattle sample (5) is "nicer" than the Scottish population (3.2) as $5 > 3.2$. 2. The scores in the Newbattle sample are more consistent (less spread out) than the population as $3.90 < 4.5$.

(c) **Effect of removing -2:**

- The **mean will increase** because we are removing the lowest number (a negative outlier), which was dragging the average down.
- The **standard deviation will decrease** because the outlier (-2) was the furthest value from the mean. Removing it makes the remaining data less spread out.

5. The 12 Days of Christmas

(a) **Table Completion:** Day 4: Gifts = 10, Total = 20. Day 5: Gifts = 15, Total = 35. (Pattern for Gifts on Day n: Triangular Numbers $\frac{n(n+1)}{2}$). (Pattern for Total T_n: Tetrahedral Numbers).

(b) **Total Calculation:** Sum of the first 12 triangular numbers: $1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78$. Total sum = **364**. (Did you know? This is one gift for every day of the year, almost!)

Questions by Bryn Jones, Newbattle High School. Find more resources at Applying Maths.

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