

Higher Apps Christmas Problems

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Questions

1. Elf Estimation

It is November 1st, and the elves have exactly 30 days to wrap all the presents for the children of the world.

Using Fermi estimation, estimate the number of elves required to complete the task.

State all assumptions made.

2. The Naughty-Nice Formula

Father Christmas uses a complex formula to calculate a child's "Goodness Index" (I). The formula is given by:

$$I = \frac{G \cdot W}{S \cdot T}$$

Where:

- G is the number of good deeds performed (count, no unit).
- W is the distance walked helping others (measured in metres, L).
- S is the speed at which chores are done (measured in metres per second, LT^{-1}).
- T is the total time spent sleeping (measured in seconds, T).

1. Identify the dependent variable in this model.
2. State the units of the variables W , S , and T .
3. Check if the formula for I is dimensionally consistent, assuming I has no units.

3. You're Sleigh Off

The sleigh travels a distance of $D = 4000$ km with an absolute error of ± 50 km. The average speed of the sleigh is $V = 800$ km/h with an absolute error of ± 10 km/h.

1. Calculate the relative error in the distance D .
2. Calculate the relative error in the speed V .
3. Estimate the total time taken ($T = D/V$) and calculate the absolute error in the time.

4. Naughty or Nice Analysis (R Studio)

The elves have organized the data into a file called `north_pole_data.csv`. The file contains two columns:

- `Boy_List`: The status of boys (Naughty/Nice).
- `Girl_List`: The status of girls (Naughty/Nice).

You can download the file [here](#).

1. Write the R code to read the CSV file into a dataframe called `north_pole_data` and use the `attach` command to make the variables accessible.
2. Write the code to produce a table of the data using the command `table(Boy_List, Girl_List)`.
3. Write the code to produce two separate bar charts: one for the Boy List and one for the Girl List.
4. An elf runs the test `prop.test(table(Boy_List, Girl_List))`. The output shows a p-value of 0.0004. Interpret this result at the 5% significance level.

5. Toy Assembly Planning

The “Super-Sled 3000” requires the following assembly tasks:

Task	Description	Duration (mins)	Predecessors
A	Carve Runners	20	-
B	Paint Runners	30	A
C	Construct Seat	40	-
D	Final Assembly	10	B, C

1. Construct a Gantt chart for this project.
2. Calculate the minimum time required to complete one sled.

6. Delivery Risk Management

Father Christmas has been awarded a contract to deliver presents to a new city. If the delivery is delayed, he faces a festive fine of £50,000. For the cost-benefit analysis, Father Christmas assumes only two events will cause a delay:

- Rudolph catching a cold (probability 0.1).
- The GPS failing (probability 0.05).

The probability of **no delay** is 0.85.

- (a) Calculate the expected cost of a delay.

Father Christmas is considering the following control measures:

- **Control Measure 1:** Buy “Reindeer Vitamins”. Cost: £2,000. Reduces probability of Rudolph’s cold to 0.02.
 - **Control Measure 2:** Install “Backup Map”. Cost: £1,000. Reduces probability of GPS failure to 0.
- (b) Calculate the expected cost (including the cost of the measure) if Control Measure 2 is taken.
- (c) The expected cost for Control Measure 1 is £3,500. Explain which control measure Father Christmas should use to minimize expected costs.

Solutions

1. Elf Estimation

Answers will vary based on assumptions. Example solution:

- **Assumptions:**

- Population of children $\approx 2 \times 10^9$.
- Gifts per child ≈ 2 . Total Gifts = 4×10^9 .
- Time to wrap one gift ≈ 5 mins (1/12 hour).
- Working hours: 30 days \times 10 hours/day = 300 hours per elf.

- **Calculation:**

- Total Hours needed = $4 \times 10^9 \times (1/12) \approx 3.33 \times 10^8$ hours.
- Number of Elves = $\frac{\text{Total Hours}}{\text{Hours per Elf}} = \frac{3.33 \times 10^8}{300}$.
- Result $\approx 1,110,000$ Elves.

2. Naughty-Nice Formula

1. The dependent variable is the **Goodness Index** (I).
2. Dimensions: $W = L$, $S = LT^{-1}$, $T = T$.
3. Consistency Check:

$$\text{RHS} = \frac{[1] \cdot [L]}{[LT^{-1}] \cdot [T]} = \frac{L}{L \cdot T^{-1} \cdot T} = \frac{L}{L \cdot T^0} = \frac{L}{L} = 1$$

The RHS is dimensionless. The LHS (I) is an index (dimensionless). Therefore, the formula is **consistent**.

3. Sleigh Navigation

1. Relative error in D : $\frac{50}{4000} = 0.0125$ (or 1.25%).
2. Relative error in V : $\frac{10}{800} = 0.0125$ (or 1.25%).
3. Total Time $T = \frac{4000}{800} = 5$ hours.
Sum of relative errors = $0.0125 + 0.0125 = 0.025$.
Absolute Error in $T = 5 \times 0.025 = 0.125$ hours.
Result: **5 ± 0.125 hours** (or 5 hours \pm 7.5 mins).

4. R Studio Analysis

1. R Code:

```
north_pole_data <- read.csv("north_pole_data.csv")
attach(north_pole_data)
table(Boy_List, Girl_List)
barplot(table(Boy_List))
barplot(table(Girl_List))
```

2. **Interpretation:** The p-value (0.0004) is less than the significance level (0.05). Therefore, we **reject the null hypothesis**. There is statistically significant evidence of a difference/association between the lists.

5. Toy Assembly Planning

1. Gantt Chart:

	0-10	10-20	20-30	30-40	40-50	50-60
Task A	█	█				
Task C	█	█	█	█		
Task B			█	█	█	
Task D						█

Table 1: Project Gantt Chart (Time in minutes)

2. **Minimum Time:** Path A-B-D: $20 + 30 + 10 = 60$ mins. Path C-D: $40 + 10 = 50$ mins. The critical path is A-B-D. Minimum time is **60 minutes**.

6. Delivery Risk Management

(a) Expected Cost (Current):

$$E(\text{Cost}) = (\text{Prob Rudolph} \times \text{Penalty}) + (\text{Prob GPS} \times \text{Penalty})$$

$$E(\text{Cost}) = (0.1 \times 50000) + (0.05 \times 50000) = 5000 + 2500 = \text{£7,500}$$

(b) Expected Cost (Measure 2 - Backup Map): New Prob(GPS) = 0.

$$E(\text{Penalty}) = (0.1 \times 50000) + (0 \times 50000) = \text{£5,000}$$

$$\text{Total Cost} = E(\text{Penalty}) + \text{Cost of Measure}$$

$$\text{Total} = 5000 + 1000 = \text{£6,000}$$

- (c) **Decision:** Measure 1 Expected Cost: £3,500. Measure 2 Expected Cost: £6,000. Father Christmas should choose **Control Measure 1** as it has the lower expected cost.

Questions by Bryn Jones, Newbattle High School. Find more resources at [Applying Maths](#).



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